

## MODERN PORTFOLIO THEORY WITH APPLICATION TO CATASTROPHE INSURANCE

In the insurance community, a portfolio refers to an ensemble of individual policies. Each policy, in turn, may cover a number of individual assets, e.g., buildings, which may or may not be spread out in various locations. In any case, a portfolio may be assumed to cover many assets in multitudes of locations. The term portfolio is not restricted to primary insurers; it applies to re-insurers as well. A re-insurer issues treaties to primary insurers, and his portfolio is made up of the portfolios of the insurers.

Insurers and re-insurers who issue catastrophe policies such as for earthquakes and hurricanes are concerned foremost with the maximum loss, usually expressed in terms of Probable Maximum Loss or PML<sup>1</sup>. The PML associated with a large portfolio can be quite substantial and can affect the solvency of a company. Hence, estimate of the PML inherent in a portfolio of policies against catastrophe, as well as other measures of risk such as average annual loss, basis risk and diversification, are a matter of great interest to insurers and re-insurers.

The risk of a portfolio is an aggregation of the risks of the member policies, which, in turn, are the aggregation of the risks at individual locations. However, the aggregation is not simply one of addition or summation. Unlike events such as fires where each accident is more or less independent from another and where the accidents are localized with respect to one another, a catastrophe casts a large foot-print which is likely to affect a large number of assets covered by a portfolio. Hence, losses to the portfolio will come from a multitude of asset/locations. Furthermore, assets that are near each other will have a much higher correlation in their damage than those far apart. The aggregation of risk for a portfolio, then, must take into consideration not only the correlation arising

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<sup>1</sup> Although a commonly used measure of risk in the insurance business, the definition of PML is far from universal. It may have very different meanings in different companies. In this paper, PML is equated with a loss threshold associated with a percentage (1%, 0.5% etc.) of being exceeded depending on the company's risk tolerance.

from occurrence of the event but also the correlation in losses between locations given that a certain event has occurred.

In modeling financial risk, J. P. Morgan has developed a methodology called *RiskMetrics* to determine the Value at Risk (*VaR*) of a portfolio<sup>2</sup>. Historical data are used to derive a covariance matrix for the correlation between investments in various sectors of the market, and this information is used to derive the portfolio variance and percentile loss. The methodology for estimating portfolio risk due to catastrophe events developed in this paper uses a similar approach. However, whereas actuarial data on investment performance may be adequate to support correlation in financial risk, such data are of limited value to catastrophe analysis for several reasons, not the least of which are the scarcity of data and the long recurrence period. We shall explain in Part I why engineering models must be used to simulate nature in estimating losses due to catastrophes, and what useful outputs for insurance decisions can be expected from the engineering approach. These quantitative measures of risk are not possible otherwise.

Inherent in the difficulty of managing catastrophe risks (and the role for insurance) is the presence of uncertainties. There are uncertainties associated with the occurrence of an event. The phenomenology models themselves are inherently uncertain, due to the scarce database and limited knowledge. These uncertainties, some epistemic and some aleatory, must be accounted for especially since the supporting database is small, and because they have strong bearing on key decision parameters such as the average annual loss and the loss exceedance probabilities. This is done in Part II.

The presence of uncertainties has two major consequences. First, they contribute to uncertainties (probabilistic variation) in the loss estimates. We emphasize this effect by referring to it explicitly as “distributed loss”, to distinguish it from its deterministic counterpart when uncertainties are ignored. Distributed loss interacts with the policy/portfolio terms and application in a manner that can be quite complex, and the allocation of the mean losses to the insured, insurer and re-insurer can be greatly affected, as will be described in Part II.

The second major consequence of uncertainties is that (distributed) losses at different locations covered by a policy/portfolio may also be correlated. We refer to this effect as loss correlation. The effect of loss correlation on policy/portfolios

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<sup>2</sup> RiskMetrics – Technical Document, Fourth Edition, Morgan Guaranty Trust Company of New York, December 1996.

is addressed in Part II and in greater detail in Part III. Loss correlation may arise due to occurrence of the event, functional dependency of the site hazards, or statistical correlation. While simulation using engineering models can be used to quantify all aspects of loss correlation rigorously, the simulation procedure though straightforward can be time-consuming and inefficient for wide application. Approximate approaches are discussed, and diversification factors are introduced in Part III as an efficient substitute for portfolio analysis.

The three main elements:

- (1) Engineering models (for quantification of physical phenomena and associated uncertainties);
- (2) Finance models (for delineation of the constraints and interactions imposed by policy/portfolio structures); and
- (3) Diversification factors (for rapid quantification of correlation among losses within the policy/portfolio)

constitute the portfolio theory that is advocated for catastrophe insurance applications.

This paper is prepared with the following audience in mind: primary insurers, re-insurers, investment companies, and security firms. For a primary insurance company, a portfolio consists of many policies with individual risks in locations that may be spread out but the risks may still be correlated. Questions of interest to insurers are: What is the probable maximum loss at a location (or the PML zones) for, say, 100 years, what is the loss ratio or average annual loss (AAL), and what are the associated confidence levels? For a reinsurance company, a portfolio consists of many cedant companies' portfolios. As much as each cedant portfolio can be assumed to have been properly quantified, questions of interest remain for a re-insurer: How are losses to the portfolios correlated, what is the probable maximum loss, and what is the loss ratio?

For strategic capital allocation, the main issue is the level of risk associated with a return. The question of interest is: Given similar levels of return, which investment has the lower level of risk? When the risk of portfolios can be properly quantified, they become an asset that can be traded in the capital market as a security. As catastrophe insurance is relatively uncorrelated with securities currently traded in the market, insurance portfolios offer a great means of diversification. The main questions are: How should insurance portfolios be securitized, and how to rate insurance portfolios?

All these issues depend on a rational way of quantifying the probabilistic loss of a portfolio given the state of knowledge and uncertainty. We shall describe step-by-step how such methodology can be developed.